Sensitivity Analysis of Induction Motor Performance

This paper, in a computer simulation of an induction motor performance from its equivalent circuit, presents the graphs of slip, speed, power factor, input power, efficiency, and stator current, against output power. As also the sensitivity values of the induction motor performance variables with respect to its equivalent circuit parameters at three different output powers. And the study reveals the effect of each circuit parameter on the power factor, speed, input power, output power, efficiency, maximum torque, stator current, and starting current respectively.

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Induction motors have gained extremely wide application in industry by virtue of the advantages they have over other types of motors. Correct selection of the power rating of induction motors is a matter of economic interest because it generally has a large bearing on the capital and running costs of numerous electric drives. A motor of insufficient power capacity may not allow the driven machine to operate satisfactorily and will lower its efficiency and output. On the other hand, use of a motor with excess power capacity reduces the economic efficiency and output. On the other hand, use of a motor with excess power capacity reduces the economic efficiency of the installation, makes it more expensive, and leads to large energy losses because of the lower efficiency of the motor at reduced loads. Furthermore, this leads to lower power factor and results in uneconomical loading of the supply circuits and generators. Accurate equivalent circuit parameters of the machine should be available for predicting its performance. While motor designers calculate the parameters of the machine using design details, measured values are preferable for prediction. Not all parameters would be constant under all operating conditions, as temperature, winding currents; saturation, skin effect etc determine their values and these have to be accounted for as far as practicable.

In designing any motor, the accuracy of its performance prediction and economy are the chief objectives. In this paper, an attempt is made to gain a deeper physical insight into the induction motor operation through sensitivity analysis of its equivalent circuit parameters. This paper carries out computer simulation of the induction motor performance from its equivalent circuit parameters, and plots the graphs of the slip, speed, power efficiency, stator current and magnetizing current against the output power it. It then carries out the sensitivity analysis of the induction motor performance variables with respect to its equivalent circuit parameters at three different output powers. The study reveals the effect each circuit parameter has on the power factor, speed, input power, output power, efficiency, maximum torque, starting torque, starting current, and magnetizing current respectively.

Performance Characteristics of the Induction Motor

The performance of the three phase induction motor from the parameters of its equivalent circuit is evaluated in the usual way. The equivalent circuit parameters R_1, R_2, X_1, X_2, $X_{\varnothing}$ and friction windage, and core losses (FWCL) are known at rated voltage and rated frequency. The following relationship for the motor holds.

$(1\text{-}s)P_g$ = Output Power + FWCL (1)

Where P_g is the gap power, (1-s) Pg is the developed mechanical power, and s is the slip.

In the familiar equivalent circuit of the induction motor, shown in Fig. 1, the impedance Z_f is the parallel combination of $[(R_2/s) + jX_2]$ and $jX_{\varnothing}$, respectively.

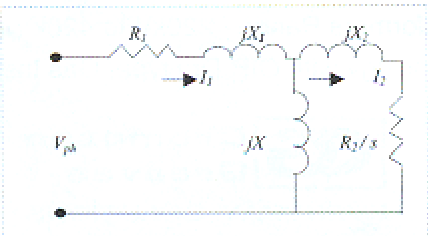


Figure 1. Equivalent circuit of induction motor

$Z_f = R_f + jX_f$ (2)

$P_g = 3I_1^2R_f$ (3)

Where I_1 is the stator current. For a given load on the motor, the slip is determined as given in the Appendix, and then the performance variables, namely, stator current, power factor, power input, efficiency, maximum torque and magnetizing current, are determined at this slip.

Sensitivity Analysis

The normalized sensitivities of the induction motor performance variables with respect to each of the equivalent circuit parameters are next determined by varying the value of one parameter at a time by 1% and determining the percentage change in the value of a particular performance variable. For example, the normalized sensitivity of the motor efficiency S_{xi}^{η} with respect to a given parameter xi is calculated from the expression

$$S_{xi}^{\eta} = (\eta_1 - \eta_0)100/\eta_0 \quad (4)$$

Where η_0 is the efficiency with nominal values of parameters and η_1 the efficiency when the value of the parameter xi is increased by 1 %. In the calculation of motor efficiency, there is some obligation to estimate how sensitive the efficiency is to small changes in the design of the motor leading from a small change in any of the equivalent circuit parameters are adjustable by varying the design of the motor leading to an increase in the efficiency of the motor. Expressions similar to (4) are applicable to other performance variables, namely, stator current, power factor, slip, power output, power input, maximum torque, starting torque, starting current, and magnetizing current. An expression similar to (4) also calculates the sensitivities of the motor performance variables with respect to the supply voltage, V and the supply frequency, f. while calculating the sensitivity of the performance variables with respect to the frequency, the values of the equivalent circuit parameters are changed proportionately with their values at the rated frequency given in Table 1. It is worth nothing that more accurate sensitivity results of the performance variables with respect to the frequency can be obtained if the equivalent circuit parameters have been measured at the required frequency.

Numerical Results

Table 1. Equivalent circuit parameters1

R_1 Ω	R_2 Ω	X_1 Ω	X_2 Ω	$X_ø$ Ω
0.294	0.144	0.503	0.209	13.25

Consider a three-phase 220 V, 7.46 kW, six-pole, 60 Hz induction motor for the study[1]. Table 1 gives the equivalent circuit parameters of the motor at 60 Hz. Results of the analytical work show that, with an increase in the motor load from 0.746 kW to 7.46 kW, the power factor and slip all increase. The magnetizing current, however, decreases as the load is increased. The efficiency and the power factor show a non-linear behavior with a change in the load on the motor. Fig.2 shows these effects.

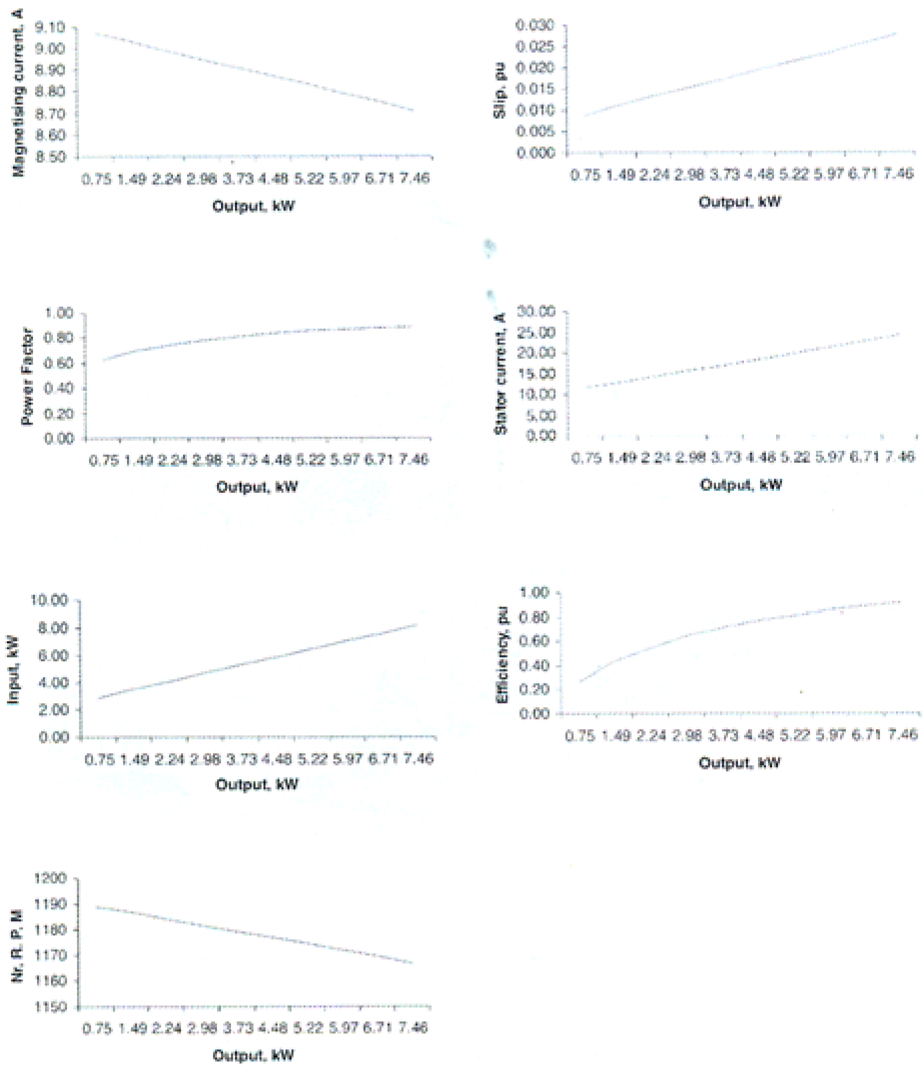
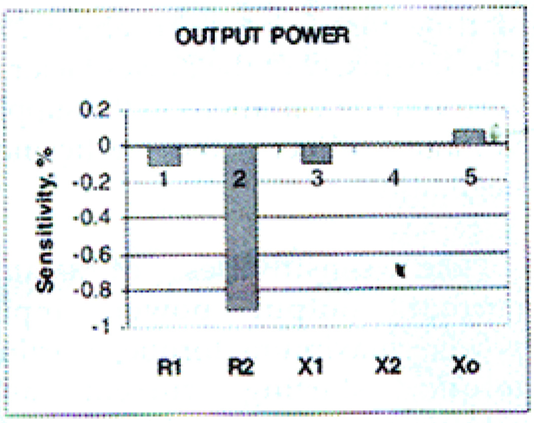


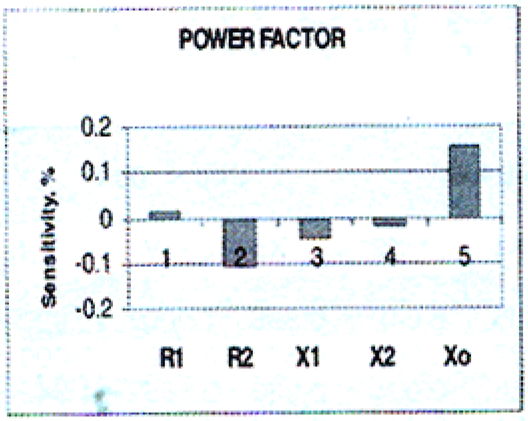
Figure 2. Performance characteristics of a 220 V, 60 Hz, 7.46 kW, three-phase induction motor

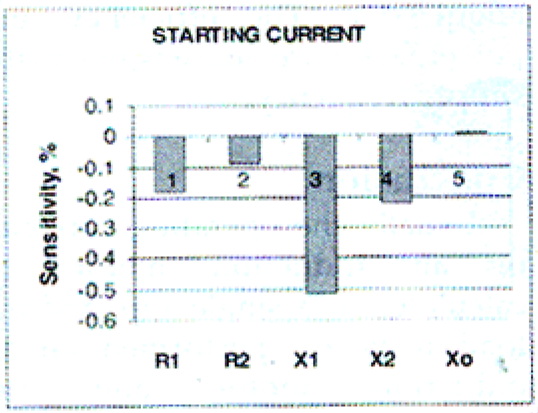
There is a correlation between the power factor of the motor and the motor efficiency such that the power factor begins to fall when the efficiency of the motor falls.

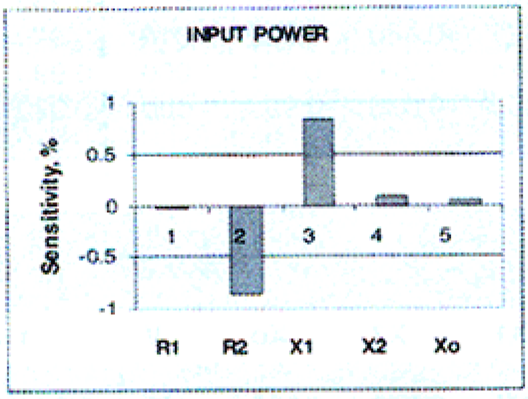
Sensitivities with respect to the Circuit Parameters

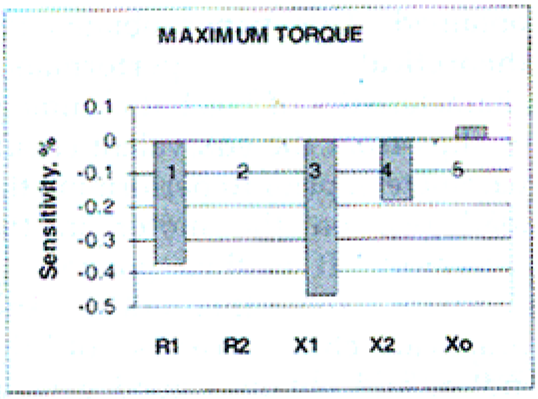
The normalized sensitivities of slip, stator current, power factor, power output, power input, efficiency, maximum torque, starting torque, starting current and magnetizing current with respect to the equivalent circuit parameters have been calculated at 10% of full-load, 70% of full-load, and 100% of full-load. Table 2 and fig. 3 give the sensitivity results. Note that the results of Fig. 3 pertain to full-load. The interpretation of the sensitivity values for the motor running at 100% full load follows:

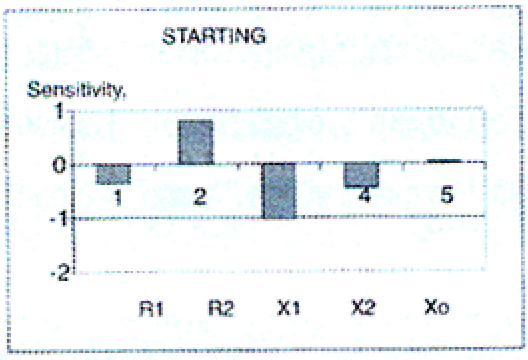

OUTPUT POWER
Sensitivity, %
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1
2
3
4
5
R1
R2
X1
X2
Xo

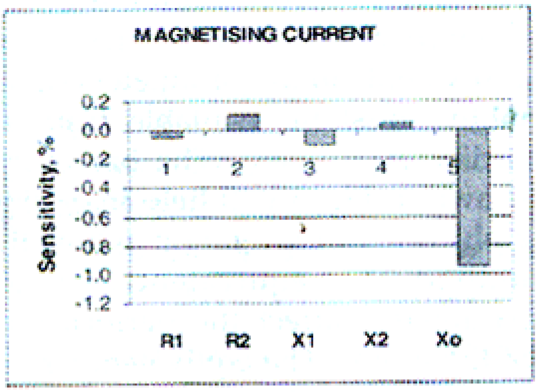

POWER FACTOR
Sensitivity, %
0.2
0.1
0
-0.1
-0.2
1
2
3
4
5
R1
R2
X1
X2
Xo


STARTING CURRENT
Sensitivity, %
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
1
2
3
4
5
R1
R2
X1
X2
Xo


INPUT POWER
Sensitivity, %
1
0.5
0
-0.5
-1
1
2
3
4
5
R1
R2
X1
X2
Xo


MAXIMUM TORQUE
Sensitivity, %
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
2
3
4
5
R1
R2
X1
X2
Xo


STARTING
Sensitivity,
1
0
-1
-2
1
2
3
4
5
R1
R2
X1
X2
Xo


MAGNETISING CURRENT
Sensitivity, %
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
1
2
3
4
5
R1
R2
X1
X2
Xo

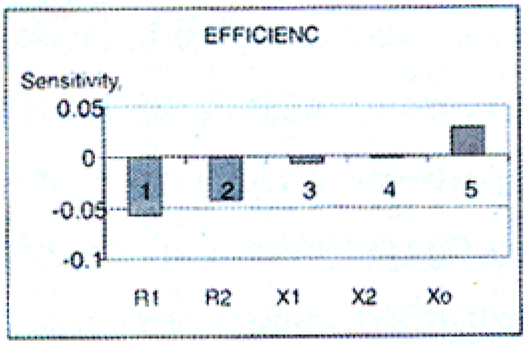

EFFICIENC
Sensitivity,
0.05
0
-0.05
-0.1
1
2
3
4
5
R1
R2
X1
X2
Xo

Table 2 Sensitivity values of the performance variables of a 220 v, 60 Hz, 7.46 kW, three-phase induction motor

Output power = One-tenth of full load = 0.746 kW					Slip = .009		
Performance variable	R1	R2	X1	X2	X0	V	f
Stator current	-0.017	-0.374	-0.356	0.042	-0.556	1.000	-0.990
Power factor	0.027	-0.557	-0.037	-0.006	0.573	0.000	0.000
Output power	-0.040	-1.122	-0.086	-0.001	0.083	2.365	-1.342
Input power	0.009	-0.955	0.881	0.073	0.020	2.010	-0.990
Efficiency	-0.050	-0.178	-0.013	-0.001	0.064	0.348	-0.355
Maximum torque	-0.374	0.000	-0.469	-0.190	0.035	2.010	-0.990
Starting torque	-0.368	0.817	-1.025	-0.440	0.022	2.010	-0.990
Starting current	-0.184	-0.091	-0.514	-0.220	0.011	1.000	-0.990
Magnetizing current	-0.017	0.035	-0.055	-0.036	-0.955	1.000	-0.990

Output power = 70% of full load =5.22 kW						Slip = .02	
Performance variable	R1	R2	X1	X2	X0	V	f
Stator current	-0.037	-0.688	0.690	0.049	-0.207	1.000	-0.990
Power factor	0.015	-0.207	-0.040	-0.013	0.242	0.000	0.000
Output power	-0.079	-0.972	-0.086	-0.006	0.073	2.165	-1.144
Input power	-0.022	-0.908	0.859	0.075	0.048	2.010	-0.990
Efficiency	-0.057	-0.042	-0.006	-0.002	0.029	0.152	-0.155
Maximum torque	-0.374	0.000	-0.469	-0.190	0.035	2.010	-0.990
Starting torque	-0.368	0.817	-1.025	-0.440	0.022	2.010	-0.990
Starting current	-0.184	-0.091	-0.514	-0.220	0.011	1.000	-0.990
Magnetizing current	-0.037	0.080	-0.083	0.038	-0.955	1.000	-0.990

Output power = full load = 0.746 kW						Slip = 0.028	
Performance variable	R1	R2	X1	X2	X0	V	f
Stator current	-0.050	-0.749	0.761	0.053	-0.115	1.000	-0.990
Power factor	0.013	-0.109	-0.044	-0.017	0.154	0.000	0.000
Output power	-0.106	-0.910	-0.093	-0.010	0.069	2.124	-1.103
Input power	-0.037	-0.870	0.827	0.080	0.056	2.010	-0.990
Efficiency	-0.069	-0.002	-0.005	-0.002	0.021	0.111	-0.114
Maximum torque	-0.374	0.000	-0.469	-0.190	0.035	2.010	-0.990
Starting torque	-0.368	0.817	-1.025	-0.440	0.022	2.010	-0.990
Starting current	-0.184	-0.091	-0.514	-0.220	0.011	1.000	-0.990
Magnetizing current	-0.050	0.114	-0.107	0.041	-0.954	1.000	-0.990

The sensitivities of the stator current and the power input with respect to R_2 and X_1 are the highest. The sensitivity with respect to R_2 is negative and that with respect to X_1 is positive. The sensitivity with respect to R_2 is the highest and negative. The values of the two parameters R_2 and X_1, however, do not have a significant influence on the motor efficiency. The sensitivities of the power factor with respect to the circuit parameters are quite low. The machine designers are, therefore, not normally interested in adjusting the machine design parameters to improve the power factor. External means, like capacitor placement, can improve the power factor. The values of R_1 and X_1 affect the maximum torque to the greatest extent. R_2 has no influence on the maximum torque. The values of R_2 and X_1 affect the starting torque most. The values of X_1 and X_2 affect the starting current the most and the value of $X_Ø$ affects the magnetizing current the most. At one-tenth of full load, in addition to the effects of R2 and X1, the value of $X_Ø$ also influences the stator current. At this load, $X_Ø$ also influences the power factor.

Sensitivities with respect to the Supply Voltage and Frequency[2]

Next, we determine the sensitivity values with respect to the supply voltage and the supply frequency. At full load, the sensitivities of the stator current, starting current and magnetizing current with respect to the supply voltage are each about 1%, and the sensitivities of input power, output power, maximum torque and starting torque are each about 2%. The sensitivity of the power factor is zero and they variations in the supply voltage affect the efficiency negligibly.

The sensitivities of stator current, output power, input power, maximum torque, starting torque, starting current and magnetizing current with respect to the supply frequency are each about -1%. The efficiency and power factor are not sensitive to the supply frequency.

Conclusion

Usually the operating variables of an induction motor are calculated at a single slip, and the load tests are performed on an induction motor and the performance characteristic similar to the ones shown in Fig. 2 are obtained. This paper obtains the theoretical performance characteristic of Fig. 2 at a number of slips from the equivalent circuit parameters, prior to performing the load test on the motor in the laboratory. It is thus possible to compare the theoretical and practical characteristics of the motor, and obtain a good general picture of the motor behavior. However, before obtaining the theoretical characteristics, the equivalent circuit parameters of the motor must be available from an earlier experiment. The bar charts shown in fig. 3 will help in getting a physical interpretation of how the individual equivalent circuit parameters affect the performance of the motor. The equivalent circuit parameters of the motor depends on the physical parameters such as length and diameter of the motor, number of poles, dimensions and numbers of the stator and rotor slots, magnetic flux density in the air gap, and the length of air gap[3]. The sensitivity studies of the operating variables of the induction motor with respect to the equivalent circuit parameters will help the design engineers to choose and adjust the physical design parameters of the motor to obtain a proper efficiency or power factor characteristic for changes in the motor load.

The sensitivity analysis has given a clear picture of how the various parameters of the induction meter equivalent circuit affect the performance variables of the induction motor. However, the paper does not take into account the nonlinearities associated with saturation and skin effects. These play an important role in the performance of the induction motor. For example, the starting torque and the starting current are strongly dependent on skin effects, while the magnetizing current is strongly nonlinear with the voltage. Therefore, further studies on sensitivity analysis of the induction motor must consider these phenomena.

Appendix

The parameters R_1, R_2, X_1, X_2 and $X_Ø$ of the induction motor equivalent circuit at the rated frequency are assumed to be known. The sum of the friction, windage and core loss, FWCL, at the rated frequency is also assumed to be known. Equation (5) given below is obtained from equations (1)-(3):

$$S=1-(\text{output power} + \text{FECL})\,(R_f+X_f^2/R_f)\,/\,V^2 \qquad (5)$$

For a given output power, the polynomial expression for the slip is derived by substituting R_f and X_f in terms of the slip and the equivalent circuit parameters. The polynomial expression is:

$$as^4 + bs^3 + cs^2 + ds + e = 0 \quad (6)$$

Where

$$a = R^2 X^2_{Ø} (X_2 + X_Ø)^2 + ((\text{output power} + \text{FWCL}) / V^2) \left(X_2^4 X_\phi^2 + 2X_2^3 X_\phi^2 + X_2^2 X_\phi^4\right)$$

$b = -R_2 X_\phi^2 \left(X_2 X_\phi\right)^2$

c = ((output power + FWCL) / V^2)

d = 0

e = ((output power + FWCL) / V^2)

The Newton-Raphson Method solves the polynomial as follows:

$$S_{i+1} = S_i + \Delta s, \qquad (7)$$

Where

$$\Delta s = -\,(a_{si}{}^4 + b_{si}{}^3 + c_{si}{}^2 + e)/(4\,a_{si}{}^3 + 3\,b_{si}{}^2 + 2_{csi}$$

By taking a suitable starting value for Si.

The stator current is then determined at this slip from (3) and the power factor and the motor input from the following equations:

$$Cos(\phi) = Cos(\tan^{-1}(X_f + X_1) / (R_f + R_1)) \qquad (8)$$

$$Input = \sqrt{3}\,V\,I_1\,Cos(\phi) \qquad (9)$$

Dr C.S. Indulkar retired from IIT Delhi as professor of Electrical Engineering in 1994, and was involved in teaching and research in electrical power systems, machines, and control systems for over 38 years. Aouthor of over 150 technical papers for some of which he received the Tata Rao Gold Medal and several Certificates of Merit. He was on the Editorial Advisory Boards and also a reviewer of papers for reputed Journals. His website http://www.geocities.com/ cindulkar – useful for instructors / students of pre-final and final year Electrical Engineering programs.

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Reference book:

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