

Power dissipation

When a current flows through a component, that component will heat up. This process is called power dissipation and is measured in Watts. The power dissipation of a device can be calculated very easily:

$$P = V \cdot I$$

Power dissipation of a resistor

Let's calculate the power dissipation of a 100 ohms resistor connected to a 9V battery.

The voltage across the resistor will be 9V. The current is $9V/100\text{ohms} = 90\text{mA}$. So the power dissipation will be: $9V \cdot 90\text{mA} = 810\text{mW}$.

It is very important to calculate the power dissipation of the components in your design. A regular resistor has a maximum dissipation rating of 0.25W (= 250mW). If you would have used such a resistor in the example above, it would have blown. A 1W resistor is a good choice.

Since it's so important, let's create an equation with which we can easily calculate the power dissipation of a resistor. We know:

$$(1) P = V \cdot I \quad (2) V = I \cdot R \quad (3) I = V / R$$

Substituting (2) in (1) and (3) in (1) respectively results in:

$$P = I^2 \cdot R \quad P = V^2 / R$$

With these equations you can easily calculate the power dissipation when you connect a DC voltage source to a resistor. But what will the power dissipation be if you connect an AC voltage source to a resistor? In that case, simply substitute V and I by the so called RMS values v_{RMS} and i_{RMS} . RMS stands for Root Mean Square. The RMS value is defined as the DC equivalent that provides the same power as the original waveform. Let's approximate the RMS value of a 1Hz sinusoidal signal with an amplitude of $1V_t$: $v = \sin(2 \cdot \pi \cdot f \cdot t) = \sin(2 \cdot \pi \cdot t)$. We take 4 samples: at 0s, 0.25s, 0.5s and at 0.75s. The values are 0, $0.5\sqrt{2}$, 1, and $0.5\sqrt{2}$. Next, calculate the square of each value: 0, 0.5, 1, and 0.5. The mean value of these squares is $(0 + 0.5 + 1 + 0.5)/4 = 2/4 = 0.5$. Finally, calculate the square root of the mean of the squares: $\sqrt{0.5} = 0.707V$. So the approximated RMS value of a $1V_t$ sinusoidal signal is 0.707V. Of course, the approximation is more accurate if you take more samples. Using some math, you can prove that $v_{\text{RMS}} = A/\sqrt{2}$ (for a sinusoidal signal). Using this equation we can calculate the RMS value of the signal of our example: $v_{\text{RMS}} = 1/\sqrt{2} = 0.707V$.

Using the theory above, we can calculate the power dissipation of a 100ohms resistor connected to a $9V_t$ sinusoidal signal: $P = v_{\text{RMS}}^2 / R = A^2 / 2R = 81/200 = 0.401W$.

Power dissipation of series-connected resistors

If you don't have a 1W resistor, and you still want to perform the experiment above, you may connect four 25ohms resistors in series. We've already learned that resistors in series act like a voltage divider: the voltage across each resistor is $9V/4 = 2.25V$. The current is still 90mA since the total resistance is the same. So each resistor dissipates $2.25V \cdot 90mA = 0.20W$. (Of course we could also use one of our 'easy' equations: $P = I^2 \cdot R = (90mA)^2 \cdot 25 = 0.20W$.)

Be carefull: always take resistors with the same resistance. Of course you could also create a 100ohms resistor with three 33ohms resistors and one 1ohm resistor in series, but you're gonna smell some smoke! Which resistor(s) will blow? The 1ohm resistor because it's the smallest? Let's see. Since we know the current is 90mA, we use the equation $P = I^2 \cdot R = (90mA)^2 \cdot 1 = 8.1mW$. The 1ohm resistor will survive! The power dissipation of each 33ohms resistor will be $(90mA)^2 \cdot 33 = 0.27W$. It may take some time, but you certainly will loose three resistors!

Power dissipation of parallel-connected resistors

Another way to create your own high wattage resistor is to connect multiple resistors in parallel. Let's create a 100ohms resistor with four parallel-connected 400ohms resistors and connect it to a 9V battery. The voltage across each resistor is 9V. So the power dissipation of each resistor is $P = V^2 / R = 9^2 / 400 = 0.20W$.

Again: always use resistors with the same resistance.